Journal of Engineering Physics and Thermophysics, Vol. 83, No. 2, 2010

PRESSURE TRANSIENT ANALYSIS OF DEFORMABLE RESERVOIRS

A. A. Shchipanov

UDC 532.546, 622.276

A new integrated methodology of evaluating the change in the permeability of a fractured or pressure-sensitive porous reservoir based on analytical or numerical simulation of a fluid inflow to a well has been proposed. The analysis draws on results of well tests. Analytical solutions of direct and inverse problems allow an evaluation of the permeability variation with decreasing pressure. Numerical simulation of an unsteady-state fluid inflow to a well (a direct problem) allows determining the diagnostic signs of variation in the reservoir permeability from the dynamics of the bottom hole pressure and an assessment of the evaluations obtained from solving an inverse problem. The methodology has been approved using results of some well tests.

Keywords: *deformable reservoir, fractured reservoir, porous medium, fluid flow (filtration), permeability variation, well tests, reservoir simulation.*

Introduction. Mathematical simulation is currently the basis for studying the structure of reservoirs of hydrocarbon fields and for assessing the efficiency of, and controlling, their production [1, 2]. The construction of a hydrocarbon reservoir model used to describe multiphase flows relies on a qualitative and quantitative estimation of the structure and properties of saturated reservoirs. The pressure variation in a reservoir has an effect on the properties of fluids and gases saturating a porous medium and on the properties of the porous medium itself, namely, porosity and permeability [1–5]. An evaluation of the porosity variation [6, 7] and allowance for it in the simulation are a prerequisite for constructing a hydrodynamic model of a reservoir in determining the elastic flow regime and hydrocarbon recovery as a result of the compressibility of a fluid–reservoir system [3, 8]. The permeability of a reservoir can vary even more than the porosity, which is characteristic of fractured carbonate and sandstone reservoirs alike [9–11]. The permeability variation appreciably affects the production of hydrocarbon reservoirs [4, 5].

There are several methods of evaluating the variation in the reservoir permeability by interpreting the results of well tests: using pressure-rate curves for reservoirs with an abnormally high pressure and the presence of fracturing [9], using pressure-rate curves with account for the pressure decrease on the external boundary of wells [10], and using pressure-buildup curves [12]. The above methods are based on the analytical solution of inverse problems for a flow of a homogeneous fluid in a homogeneous porous medium.

It should be noted that transport and storage properties of a reservoir are determined by its effective stress [12], that is, by the difference between the total stress in the reservoir and the pressure of fluids saturating it. In turn, the total stress in a reservoir is formed under the action of its overburden and the pressure of the fluids saturating the reservoir. Since the transient response of pressure (pressure evolution) causes a change in the effective (and total) stress in a reservoir, in the interpretation of well tests and in the simulation of the reservoir production pressure can serve as an "independent" parameter that determines the variation in transport and storage properties of a reservoir.

The present study is devoted to developing approaches to the evaluation of the variation in the permeability of fractured and pressure (stress)-sensitive reservoirs of hydrocarbon fields based on analytical and numerical simulation of a fluid inflow to a well, which allows an estimation of the pressure dependence of the reservoir permeability and a direct hydrodynamic simulation.

Well Tests. Among the most productive methods for studying the structure of reservoirs of hydrocarbon fields and assessing their transport and storage properties is a well test [6–8]. The analysis of results of well tests allows as-

---

Perm State University, 15 Bukirev Str., Perm, 614990, Russia; email: schipanov@psu.ru. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 83, No. 2, pp. 235–247, March–April, 2010. Original article submitted October 16, 2008; revision submitted May 5, 2009.

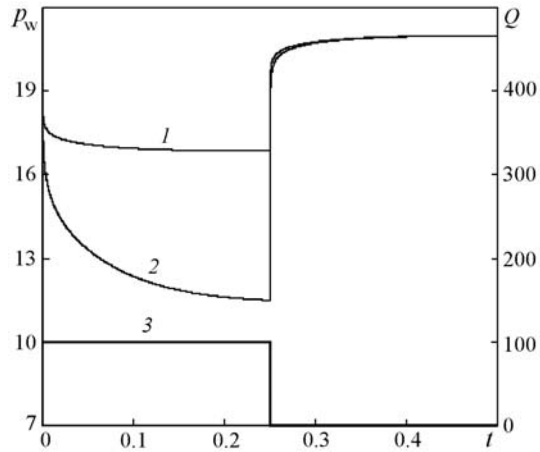


Fig. 1. Well test with an opening with a constant flow rate (Q = 100 m^3/day), a subsequent shut-in (t = 0.25 day), and a response of the bottom hole pressure p_w (pressure drawdown and buildup curves): 1) response of the bottom hole pressure p_w at the constant permeability k = 0.3 μm^2, 2) same, at pressure-dependent permeability (k^0 = 0.3 μm^2 and α_r = 0.3 MPa^{-1}), and 3) flow rate Q. p_w, MPa; Q, m^3/day; t, day.

certaining the specific features of the structure of a reservoir, namely, the presence of faults and a fracture network, its linkage to other reservoirs, and reservoir properties such, as permeability and porosity, and their reservoir-pressure dependence. The analysis is based on solving inverse (ill-posed) problems that have no unique solution and on applying expert estimates of an analyst. The estimates should be substantiated by results of additional investigations (geophysical, reservoir, and laboratory ones).

The considered well tests (Fig. 1) envisage an opening of a well with a certain flow rate at its wall followed by its shut-in [6–8]. During the operation of a well with a certain flow rate (or during its operation in a certain regime dependent on the choke diameter), the decreasing bottom hole pressure is measured (which can be regarded as pressure on the well wall). A subsequent shut-in of a well (a closing at the well head or bottom hole) causes the bottom hole pressure to increase, which is also recorded. Thus, a well test consists of consecutive steps of pressure increase and decrease in the near well-bore area of a reservoir and on the well wall, which is reflected by curves of the drawdown and buildup of the bottom hole pressure (Fig. 1). Several such consecutive steps of the well operation in various regimes are possible, with pressure-buildup periods between these steps either present or absent.

The analysis of well tests can be subdivided into two levels [6–8], namely, qualitative and quantitative. A qualitative evaluation based on the pressure-decrease (buildup) curves enables a determination of structural features of a reservoir, and a quantitative evaluation allows an assessment of the reservoir properties. In either case use is made of various transformations of the initial curves of the pressure variation and of the calculated first-order time derivatives of pressure relying on their restructuring in new coordinate systems (semi- and bilogarithmic using natural and decimal logarithms). The transformed curves are applied to solving inverse problems of finding coefficients of equations with a known solution (pressure).

Until recently inverse problems of this kind were solved mostly analytically. However, simplifications in the problem statement needed for an analytical solution hinder an assessment of some reservoir properties. This shortcoming has recently been eliminated by extensively using numerical methods that make it possible to solve the problem as stated initially and therefore, to evaluate the entire set of its parameters. The most efficient is the combination of an analytical technique and numerical procedures. The first allows a rapid evaluation of a part of the reservoir parameters and an approximate estimate of some of the remaining ones. Numerical simulation can be a method for both assessing the adequacy of analytical solutions and determining the problem parameters yet to be found.

The current study develops the approach of simultaneous use of analytical solutions and numerical simulation for interpreting results of tests of wells that drain reservoirs with pressure-sensitive permeability.

Analytical Models of a Fluid Inflow to a Well. A steady-state radial fluid inflow to a well from the external boundary can be described using the known Dupuit's formula [3, 6, 13] based on the combination of Darcy's law [1, 3, 6] and the law of mass conservation:

$$Q = I_p (p_e - p_w)\,, \quad I_p = \frac{2\pi h k}{\mu \ln (r_e/r_w)}\,.$$

It should be noted that this representation is suggestive of constant pressure on the wall of a well and its external boundary.

If the permeability of a porous medium is assumed to be pressure-dependent [4, 5, 9–11]

$$k(p) = k^0 \exp(\alpha_r [p - p^0])\,, \tag{1}$$

it is possible to obtain an expression for the flow rate for a well (an analog of Dupuit's formula) for a deformable porousmedium

$$Q = \frac{I_p^0}{\alpha_r} \left\{ \exp(\alpha_r [p_e - p^0]) - \exp(\alpha_r [p_w - p^0]) \right\}, \quad I_p^0 = \frac{2\pi h k^0}{\mu \ln (r_e/r_w)}\,. \tag{2}$$

Under the assumption of the equality of pressures $p^0 = p_e$, expression (2) simplifies to

$$Q = \frac{I_p^0}{\alpha_r} [1 - \exp(-\alpha_r \Delta p)]\,,$$

where $\Delta p = p_e - p_w$.

In the case with constant permeability (and viscosity) of a reservoir it is possible to find an analytical solution of the problem of transient response of pressure (the pressure evolution) in it under the action of a source (a well) with a specified intensity [3, 13]. The transient response of pressure can be defined by a piezoconductivity equation with initial and boundary conditions

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial p}{\partial r} \right) = \frac{1}{\kappa} \frac{\partial p}{\partial t}\,, \quad p(r, 0) = p(r, t)\big|_{r \to \infty} = p_e\,, \quad r \frac{\partial p}{\partial r}(r, t)\bigg|_{r \to 0} = \frac{Q}{2\pi\varepsilon}\,, \tag{3}$$

where

$$\kappa = \frac{k}{\phi \mu [c_r + c_f]}\,; \quad \varepsilon = \frac{kh}{\mu}\,.$$

The solution of system (3) is

$$p(r, t) = p_e - \frac{Q}{4\pi\varepsilon} \left[-\mathrm{Ei} \left(\frac{r^2}{4\kappa t} \right) \right]. \tag{4}$$

Solution (4) allows a survey of the pressure variation on the well wall and in the near well-bore area of a reservoir, and can also be used for evaluating the size of the depression funnel (of the near well-bore area of a reservoir where pressure has changed relative to its initial distribution) and therefore, the external radius of a well. The radius r_e can be approximately evaluated with accuracy δ using the relation that follows from solution (4):

$$\frac{Q}{4\pi\varepsilon} \left[-\mathrm{Ei} \left(\frac{r_e^2}{4\kappa t} \right) \right] \le \delta \, |p_e - p_w|\,. \tag{5}$$

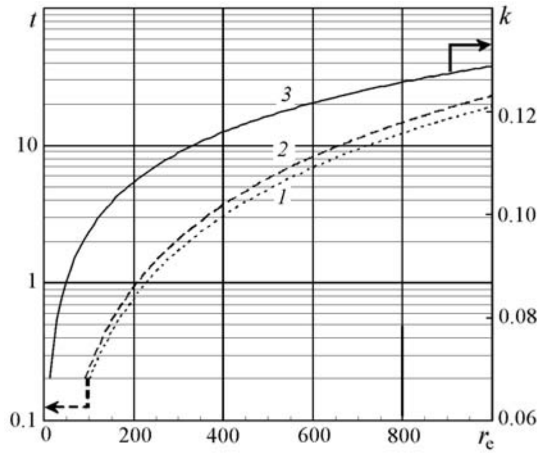


Fig. 2. Increase in the external radius of a well r_e (an enlargement of the depression funnel) over time for constant permeabilities $k = 0.127$ (1) and 0.106 μm^2 (2), $\delta = 1 \cdot 10^{-2}$; the permeability k as a function of the external radius (3). t, day; k, μm^2; r_e, m.

Figure 2 shows curves for the increase in the external radius of a well at various reservoir permeabilities, which are constructed on the basis of relation (5) for various times t.

The obtained analytical solutions are subsequently used in solving the inverse problem of evaluating the variation in the reservoir permeability and in estimating the external radius of a well, which determines the dimension of the simulation area of the numerical model of an inflow to a well (when boundary conditions are established).

Numerical Simulation of an Inflow to a Well. Results of the analysis of well tests can be used for simulating the reservoir hydrodynamics, specifically, for describing a multiphase fluid flow originating in the development of reservoirs of hydrocarbons fields with the aid of a well system. Numerical models with a radial coordinate and small blocks of a difference grid (which is refined near a well) fairly accurately describe processes in the near-well region of a reservoir. However, the estimates of the reservoir properties obtained on these models need to be corrected for use in the reservoir model with a Cartesian grid (or a grid with geometry of the angular point [14]) and with relatively large blocks. In the present study an inflow to a well is calculated using a rectangular Cartesian grid in order to make the conversion from a radial difference grid (the correction of parameters) unnecessary.

Numerical simulation of well tests draws on a simulation tool [15] that allows a simulation of two-dimensional single- and two-phase flows in a reservoir with a well system. A single-phase fluid flow is described by the following equation with initial and boundary conditions [1, 3, 15]:

$$\frac{\partial}{\partial x}\left[\frac{kh}{\mu B}\frac{\partial p}{\partial x}\right] + \frac{\partial}{\partial y}\left[\frac{kh}{\mu B}\frac{\partial p}{\partial y}\right] = \frac{\phi h}{B}\,[c_r + c_f]\,\frac{\partial p}{\partial t} + q\,, \tag{6}$$

$$p\,(x, y, 0) = p^0\,(x, y)\,, \tag{7}$$

$$p\,(x, y, t)\big|_{\Omega} = p_{\Omega}\,(x, y)\,. \tag{8}$$

The pressure variation of all these parameters (the reservoir and liquid compressibility) is allowed for by exponential functions similar to (1). It should be noted that Eq. (6) is nonlinear, since the permeability, as well as the viscosity and the volume coefficient, depend on pressure. The inflow (drain) to a well is simulated by means of a point source q in Eq. (6). The flow rate of a well is calculated using an approach [1] that suggests that a matched expression similar to Dupuit's formula be employed on the difference grid:

$$q = \frac{1}{\Delta x \Delta y} \frac{2\pi kh\,(p_w - \bar{p})}{\mu B \ln\left(\frac{1}{r_w}\sqrt{\frac{\Delta x \Delta y}{\pi}} - 1.037\right)}.$$

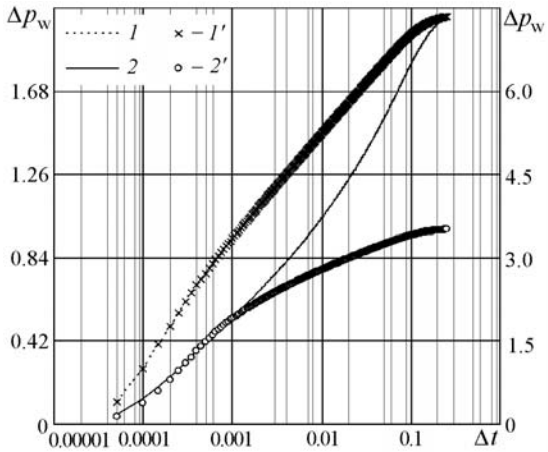
Analysis of Well Tests for Evaluating the Variation in the Reservoir Permeability. The current study aims at developing a procedure for evaluating the variation in the reservoir permeability using analytical solutions and numerical simulation. The specifics of evaluating the pressure-dependent permeability is the unfeasibility of finding an exact analytical solution of the evolution equation for pressure. Under these conditions numerical simulation allows an analysis of dynamics of the bottom hole pressure. Nonetheless, an exact solution for pressure is possible under the assumption that the fluid inflow to a well from the external boundary is steady-state, which will be used in a subsequent analysis. The methods of analysis of well tests examined below envisage an imposition of the following restrictions on initial data.

It is assumed that a single-phase fluid flow arrives from the external boundary to the wall of a well. Thus, consideration is given to an inflow of a homogeneous fluid (oil or water) to a well. The presence of a small portion of water in the oil flow (a water flooding of up to 5%) is possible. The condition of a single-phase flow suggests that during tests the bottom hole pressure does not decrease below the pressure of oil saturation with gas, which is usually present in a dissolved state. It is also assumed that the pressure on the external boundary of a well, which is equal to the initial reservoir pressure, is constant. Numerical simulation admits the establishment of conditions of specified pressure or flow rate (of a no-flow condition) on the boundary of the simulation area.

Numerical simulation: diagnostic signs of a variation in the reservoir permeability. The first component of the proposed integrated methodology is an analysis of well tests based on synthetic curves of transient response of pressure obtained using numerical simulation. The purpose of an analysis is to detect diagnostic signs of the pressure variation from dynamics of the bottom hole pressure. Let us consider the pressure transient response of pressure in a reservoir with constant and pressure-dependent permeability ($k = k^0 = 0.3$ μm^2 and $\alpha = 0.3$ MPa^{-1}). The condition of constant pressure equal to the initial pressure $p(x, y, t)\big|_\Omega = p(x, y, 0) = 21$ MPa is established on the boundary of the simulation area (any pressure losses in this area are compensated by the inflow from the outside). The well operation in one regime is simulated (Fig. 1), namely, a pressure drawdown for 6 h caused by the opening of a well with flow rate $Q = 100$ m^3/day followed by the well shut-in ($Q = 0$ m^3/day) for the same period of time with a pressure buildup. The well operation causes a pressure decrease to 16.9 and 11.5 MPa in the case of constant and pressure-dependent permeability, respectively. In the last case a threefold decrease (up to 0.1 μm^2) in the permeability in the grid blocks is observed.

An analysis of the pressure-difference curves in semilogarithmic coordinates (using a decimal logarithm) reveals the first diagnostic sign of the variation in the reservoir permeability (Fig. 3): a downward convexity of the curve of transient response of pressure in the case of pressure drawdown and an upward convexity in the case of pressure buildup. Thus, at pressure-dependent permeability the pressure-difference curves are essentially different: at pressure drawdown and buildup they are convex toward different axes, thereby forming a γ-shaped diagram. At constant permeability the curves for pressure drawdown and buildup coincide and are slightly convex upward (Fig. 3). It should be noted that the coincidence of the curves of pressure drawdown and buildup is due to the boundary condition of constant pressure on the external boundary. With other boundary conditions (a specified flow rate or an infinite reservoir), the drawdown of pressure differs from its buildup at constant permeability also.

A more vivid sign of the variation in the reservoir permeability is observed in the analysis of the pressure derivative in bilogarithmic coordinates (Fig. 4). The curves for the pressure difference and derivative of pressure with its decrease and increase coincide in the case of constant permeability. This indicates that the pressure drawdown and buildup are in this case equivalent. The difference in the curves of the pressure drawdown and buildup, which was noted above, is more clearly expressed in the pressure derivative (in the time interval from 10^{-3} to 10^{-1} day): the pressure decrease is characterized by a segment of the increase in the value of the pressure derivative, and the pressure increase is defined by a segment of the decrease in the value of the pressure derivative. At the same time, at constant permeability the pressure derivative remains unchanged in this time interval with decreasing and increasing pressure. Constancy of the pressure derivative is determined by the so-called infinite acting radial flow characterized by the ab-



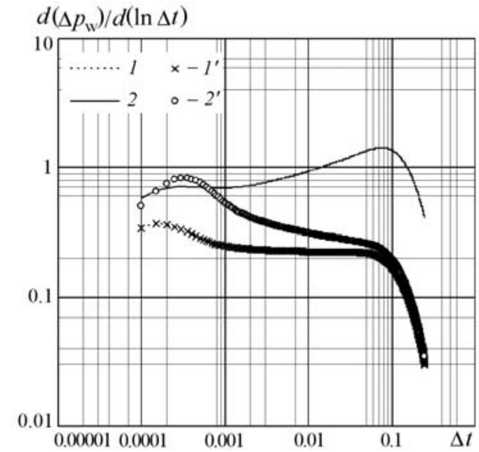


Fig. 3. Curves of drawdown (1, 2) and buildup (1′, 2′) of the pressure p_{w} at a constant (1, 1′) and pressure-dependent (2, 2′) permeability k in semilogarithmic coordinates. Δp_{w}, MPa; Δt, day.

Fig. 4. Curves of the derivative for drawdown (1, 2) and buildup (1′, 2′) of the pressure p_{w} at a constant (1, 1′) and pressure-dependent (2, 2′) permeability k in bilogarithmic coordinates. $d(\Delta p_{\mathrm{w}})/d(\ln \Delta t)$, MPa; Δt, day.

sence of the influence of the well bore and boundary effects in the reservoir [8]. The presence of the segment of a constant pressure derivative is the condition for determining constant permeability of a reservoir [8]. The last clearly defined segment of a decrease in the pressure derivative for all curves is specified by the boundary condition of constant pressure.

The noted behavior of the pressure derivative in bilogarithmic coordinates and the γ-shaped diagram in semilogarithmic coordinates are characteristic signs of the variation in the reservoir permeability caused by a pressure change. It should be pointed out that the pressure also affects fluid properties changing, as a rule, to a smaller degree ($\alpha_{\mathrm{r}} \sim 1 \cdot 10^{-1}$ MPa^{-1} [4, 5, 9–11] and $c_{\mathrm{r}} + c_{\mathrm{f}} \sim 1 \cdot 10^{-3}$ MPa^{-1} [1, 3]). Thus, a pressure decrease causes the fluid density and viscosity to decrease. Therefore, the signs considered above can also characterize the variation in the fluid density caused by a pressure change [8]. On the other hand, a variation in the fluid viscosity will camouflage the revealed effects in the pressure behavior.

A decrease (an increase) in the pressure derivative can also be caused by specific features of the near well-bore area, such as a spatial nonuniformity in terms of permeability, the presence of conducting (nonconducting) faults, hydrofractures, etc. [8]. However, their influence does not depend on dynamics (the direction of change) of the pressure. Therefore, effective practical application of the revealed diagnostic signs is possible when the curves of drawdown and buildup of the bottom hole pressure are compared.

Analytical models: identification of the variation in the reservoir permeability. The second component of the integrated methodology is the estimation of the reservoir permeability based on analytical solutions for the fluid inflow from the external boundary to the wall of a well, which necessitate certain assumptions. The assumption of a steady-state inflow allows obtaining an exact analytical solution for a reservoir with varying permeability. This forms a basis for solving the inverse problems of finding the permeability from the results of well tests (being ill-posed, this problem does not have a unique solution [16]). Initial data for the inverse problem are the measured pressure and flow rate on the well wall averaged for constructing a pressure-rate curve, which reflects a steady-state inflow to a well at various reservoir-well pressure drops [9, 10, 17]. A reservoir with constant permeability (fluid conductivity) is characterized by a linear dependence of flow rate on bottom hole pressure. The upward convexity of the curve for a producer well (a downward convexity for an injector well) can serve as a sign of the variation in the reservoir permeability [9, 10, 17]. The curve convexity can also be associated with the presence of inertial resistances arising at high fluid flow rates, with a deterioration of properties of the near well-bore area of a reservoir, with a change in the working thickness of the reservoir, etc. [17, 18].

The solution of the problem of identifying the permeability variation was proposed by A. T. Gorbunov and V. N. Nikolaevskii [9, 17] for fractured reservoirs and an abnormally high initial reservoir pressure (much higher than the hydrostatic pressure). We now consider the case with two points (flow rate and bottom hole pressure) on the pressure-rate curve, which allows an evaluation of the permeability variation.

The Gorbunov–Nikolaevskii method [9]. The method is based on calculating the area under the flow curve for the pressure-rate curve (relation (2) is used)

$$S = \int_0^{\Delta p} Q d(\Delta p) = \frac{I_p^0}{\alpha_r} \int_0^{\Delta p} [1 - \exp(-\alpha_r \Delta p)]\, d(\Delta p) = \frac{I_p^0}{\alpha_r} \left[\Delta p - \frac{1 - \exp(-\alpha_r \Delta p)}{\alpha_r} \right].$$

In the same manner, it is possible to calculate the area of the rectangle

$$\bar{S} = Q \Delta p = \frac{I_p^0}{\alpha_r} [1 - \exp(-\alpha_r \Delta p)]\, \Delta p\, .$$

The quotient of these areas

$$\frac{S}{\bar{S}} = \frac{1}{1 - \exp(-\alpha_r \Delta p)} - \frac{1}{\alpha_r \Delta p}$$

depends only on $\alpha_r \Delta p$, which, with the pressure drop Δp known, makes it possible to evaluate α_r from the relation

$$\frac{1}{1 - \exp(-\alpha_r \Delta p)} - \frac{1}{\alpha_r \Delta p} - \frac{S}{\bar{S}} = 0\, . \quad (9)$$

The solution of Eq. (9) for $\alpha_r \Delta p$ can be found, for example, using the method of bisecting the segment [19] (a tabulation of the function was proposed in [9, 17]). It should be noted that the solution for α_r does not depend on the productivity I_p^0 but instead is entirely determined by the convexity of the flow-rate curve.

We next consider the problem of identifying the variation in the reservoir permeability (of finding the coefficient α_r) using the method of [9] from two known values of the flow rate and the corresponding bottom hole pressure

$$Q_{i=0}\,(p_{w,i=0} = p_e) = 0\, , \quad Q_i\,(p_{w,i}) > 0\, , \quad i = 1, 2\, . \quad (10)$$

Let us denote

$$Q_i = \frac{I_p^0}{\alpha_r} [1 - \exp(-\alpha_r \Delta p_i)]\, , \quad i = 1, 2\, , \quad (11)$$

where $\Delta p_i = p_e - p_{w,i}$. The area under the flow curve, which is calculated using the trapezium method, and the area of the rectangle containing this curve are defined as

$$S = \sum_{i=1}^{2} \frac{Q_i + Q_{i-1}}{2} (p_{w,i} - p_{w,i-1})\, , \quad \bar{S} = Q_2 \Delta p_2\, .$$

The solution of the corresponding equation (9) allows one to find the permeability modulus α_r, which gives us the opportunity to evaluate the permeability at various pressures. The permeability at the initial pressure k^0 can be calculated from relation (11) written for each of the two points (flow rate and bottom hole pressure) on the pressure-rate curve

$$k^0 = \frac{Q_i \alpha_r \mu \ln (r_e/r_w)}{2\pi h \, [1 - \exp(-\alpha_r \Delta p_i)]}, \quad i = 1 \quad \text{or} \quad i = 2 .$$

Thus, depending on the choice of the reference point on the pressure-rate curve it is possible to obtain two different estimates for k^0.

The flow rate and the productivity at the bottom hole pressure p_i are related (at $p^0 = p_{w,i}$ in Eq. (2)) as

$$Q_i = \frac{I_{p,i}}{\alpha_r} [\exp(\alpha_r \Delta p_i) - 1] , \quad I_{p,i} = \frac{2\pi h k_i}{\mu \ln (r_e/r_w)} , \quad i = 1, 2 . \tag{12}$$

On the basis of relation (12) it is possible to evaluate the permeability k_i at the pressure p_i:

$$k_i = \frac{Q_i \alpha_r \mu \ln (r_e/r_w)}{2\pi h \, [\exp(\alpha_r \Delta p_i) - 1]} , \quad i = 1, 2 .$$

The permeability at arbitrary pressure from the considered interval can be estimated from the expressions

$$k(p) = \frac{Q_i \alpha_r \mu \ln (r_e/r_w)}{2\pi h \, [\exp(\alpha_r (p_e - p)) - 1]} , \quad i = 1 \quad \text{or} \quad i = 2 . \tag{13}$$

Depending on the choice of the reference point i on the pressure-rate curve, two permeability functions of pressure are obtained that are determined using expressions (13). They offer the possibility of calculating the flow rate from expression (12) for each value of the pressure drop, i.e., of solving the direct problem. The first function provides correspondence of the initial and calculated flow rates at the first point of the pressure-rate curve, and the second function, at the second point.

The area under the flow curve is calculated using the trapezium method [9], which leads to an understatement of the area because of the linear interpolation between the points of the flow-rate curve. This, in turn, understates the value of the sought modulus α_r. Accuracy of the solution can be improved by more correctly calculating the area of the flow curve using, for example, an approximation of the experimental points by a polynomial.

The performed analysis reveals two specific features of the method of [9]; the dependence of the solution on the reference points (with respect to which the permeability function of pressure is constructed) and the understatement of the coefficient α_r. We next consider a new approach that allows a more accurate evaluation of α_r using an unambiguously defined procedure.

A new method of evaluating the permeability variation. The initial reservoir pressure and the bearing pressure at the external boundary are assumed to be equal, which leads to a simplified expression for the flow rate (11). The problem of identifying α_r from known values of the flow rate and bottom hole pressure (Eq. 10), which was considered above, can be reduced to finding a solution of the equation

$$f(\alpha_r) = \frac{1 - \exp(-\alpha_r \Delta p_1)}{Q_1 \alpha_r} - \frac{1 - \exp(-\alpha_r \Delta p_2)}{Q_2 \alpha_r} = 0 . \tag{14}$$

The function $f(\alpha_r)$ monotonically decreases and has a single root. Equation (14) can be solved using the method of segment bisection [19] or any other method of approximate solution. The permeability at arbitrary pressure can be calculated using expression (13). Here the proposed approach provides a single pressure dependence of permeability independent of the choice of the reference point.

In the general case the solution of the inverse problem reduces to finding the root of the equation

$$f(\alpha_r) = \frac{\exp(\alpha_r [p_e - p^0]) - \exp(\alpha_r [p_{w,i} - p^0])}{Q_i \alpha_r} - \frac{\exp(\alpha_r [p_e - p^0]) - \exp(\alpha_r [p_{w,i+1} - p^0])}{Q_{i+1} \alpha_r} = 0 ,$$

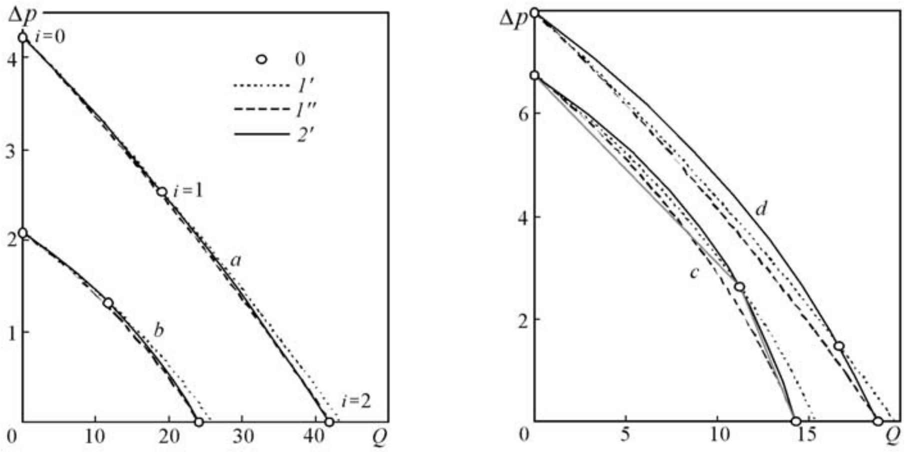


Fig. 5. Pressure-rate curves for four wells a–d: 0) based on well tests; 1′, 1″, and 2′) based on results of the simulation using a method [9] (1′, 1″) and the proposed method (2′). $\Delta p = p_w - p_{w,2}$, MPa; Q, m^3/day.

TABLE 1. Results of Interpretation of Pressure-Rate Curves a–d (Figs. 5 and 6): 1, method [9]; 2, proposed method

Pressure-rate curve	Method of interpretation	Reference regime	Designation in figures	α_r	k^0
	1	1	*1′*	0.078	0.135
a	1	2	*1″*	0.078	0.131
	2	1, 2	*2′*	0.107	0.138
	1	1	*1′*	0.343	0.038
b	1	2	*1″*	0.343	0.036
	2	1, 2	*2′*	0.477	0.040
	1	1	*1′*	0.156	0.063
c	1	2	*1″*	0.156	0.058
	2	1, 2	*2′*	0.241	0.073
	1	1	*1′*	0.055	0.039
d	1	2	*1″*	0.055	0.037
	2	1, 2	*2′*	0.139	0.050

where the bearing pressure can be other than the initial reservoir pressure and the number of points on the pressure-rate curve can be larger than 2.

The above theory has been tested on the data for four wells a–d (Fig. 5, see Table 1). Figures 5 and 6 present initial pressure-rate curves based on the results of tests of wells a–d (Fig. 5); results of their analysis using the two described methods, where 1 is the method of [9] and 2 is the proposed method (Fig. 6, see Table 1); and synthetic pressure-rate curves constructed on the basis of calculation of the flow rate from expression (12) using the obtained pressure dependences of permeability (13) (Fig. 5). Graphs of the function $f(\alpha_r)$ obtained in the analysis of the pressure-rate curves from Fig. 5 are presented in Fig. 7. The use of the method of [9] provides coincidence of the initial and synthetic flow rates at the point of the pressure-rate curve taken as the reference one in the analysis (for example, at the point $i = 1$). A marked deviation of the synthetic curve from the initial one is observed at the second point $i = 2$ (Fig. 5). It should be noted that the error of the method of [9] becomes larger with increasing convexity of the pressure-rate curve (Fig. 5, see Table 1). The proposed method provides coincidence of the initial and calculated flow rates at both points ($i = 1, 2$), which makes it possible to improve the accuracy of determining the permeability variation.

A comparison of the pressure-rate and permeability curves (Figs. 5 and 6) also allows one to conclude that insignificant deviations of the flow rate lead to appreciable deviations of the permeability. From this it follows that the procedure of averaging and the accuracy of measuring the flow rate (the bottom hole pressure) markedly affect the estimate of the permeability and its variations.

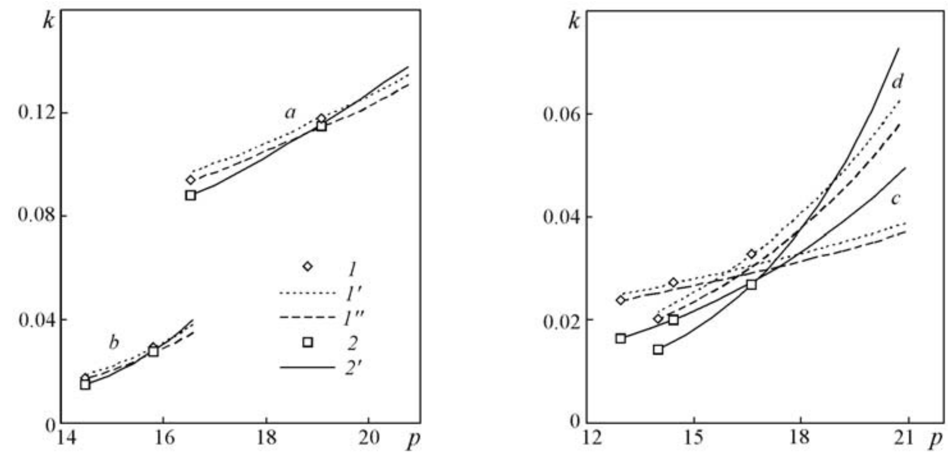


Fig. 6. Permeability variation obtained in analyzing the pressure-rate curves a–d in Fig. 5 using the method [9] (1, 1′, 1″) and the proposed method (2, 2′). *k*, μm^2; *p*, MPa.

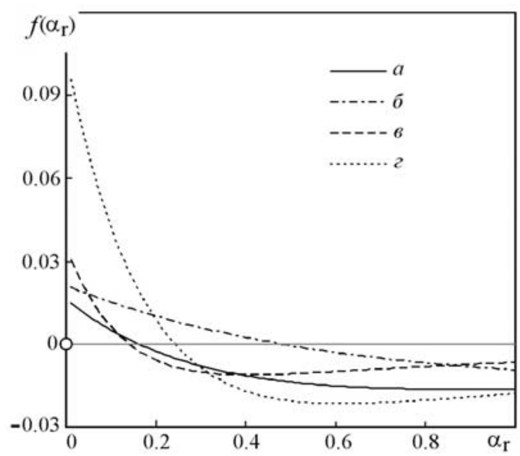


Fig. 7. Function used in evaluating the permeability modulus by the proposed method. *f*, MPa·day/m^3; α_r, MPa^{-1}.

Evaluation of the variation in the reservoir permeability based on the integrated methodology. The pressure dependence of permeability is identified by simultaneously applying three components; diagnostic signs of the permeability variation that are obtained using numerical simulation, an analytical solution that allows an evaluation of the permeability variation with a steady-state flow, and numerical simulation of initial tests of wells (of an unsteady-state flow) for approving the obtained analytical solution and, if needed, refining it.

Let us examine the use of the integrated methodology as exemplified by the test of the well *a*, for which the pressure-rate curve and permeability functions are presented in Figs. 5 and 6. The pressure-rate curve is slightly convex toward the flow-rate axis, which explains the a slight pressure dependence on permeability. It should be pointed out that the convexity of this curve can be linked with the error of measurements and their averaging. However, in contrast to other wells, there is a sufficient number of measurements of the bottom hole pressure for this well (Fig. 8), which allows a comparison of the pressure-drawdown and buildup curves, as well as the pressure measurements, with the results of numerical simulation. Thus, an analysis of this well test allows an assessment of all the three components of the developed methodology.

The test of the well *a* consists of three regimes (Fig. 8): the regime A, which is the well operation with a flow rate of 19 m^3/day for 25 h; the regime B, which is the well shut-in for 47 h; and the regime C, which is the well opening for 29 h with a flow rate of 42 m^3/day. An analysis of the curves of the pressure drawdown (the regime C) and buildup (the regime B) in semilogarithmic coordinates (Fig. 9) reveals a downward convexity of the pres-

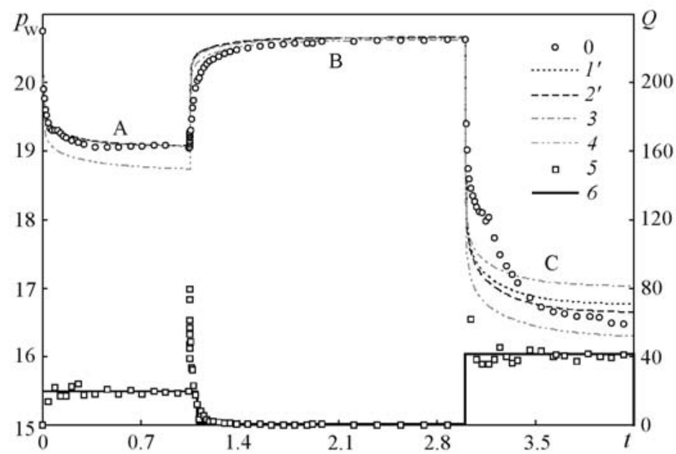


Fig. 8. Test of the well a: pressure measurements (0) and results of numerical simulation (1′, 2′, 3, 4); measurements of the flow rate (5) and its averaging (6): 1′) permeability obtained using the method [9], 2) permeability obtained using the proposed method, 3) constant permeability k = 0.127, and 4) k = 0.106 μm^2. p_w, MPa; Q, m^3/day; t, day.

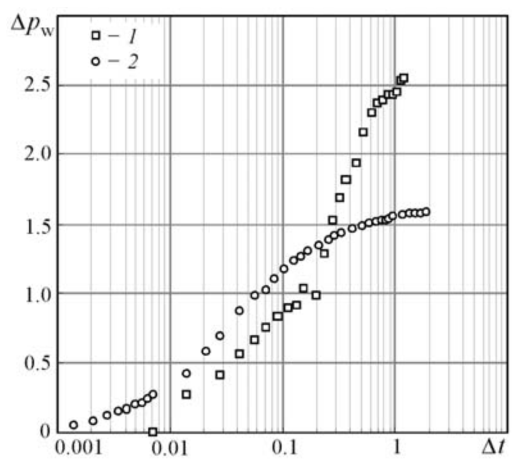


Fig. 9. Transient response of the pressure p_w in semilogarithmic coordinates for testing the well a: 1) pressure drawdown (the regime C in Fig. 8); and 2) pressure buildup (the regime B in Fig. 8). Δp_w, MPa; Δt, day.

sure-drawndown curve downward and an upward convexity of the pressure-buildup curve, which is characteristic of reservoirs with pressure-dependent permeability (Fig. 3). Because of the small number and high noisiness of the pressure measurements it is impossible to construct a fairly smooth pressure-derivative curve for detecting diagnostic signs (Fig. 4).

The conclusive stage of each regime (Fig. 8) is characterized by stabilization of the bottom hole pressure (this is more typical of the regimes A and B), which can be viewed as a sign of the flow approaching a steady state. On the basis of the flow rate and bottom hole pressure average for each regime at the concluding stage, it is possible to construct the pressure-rate curve (Fig. 5, the well a). The use of two methods of interpreting the pressure-rate curve allows one to obtain three pressure dependences of permeability (Fig. 6, the well a).

Numerical simulation of the well test is performed for estimating the accuracy of the obtained dependences. For the calculations use is made of a two-dimensional model of a reservoir. The size of the simulation area is chosen proceeding from analytical evaluation (5) of the external radius of the well. This radius, used in solving the inverse and direct problems, is a quantity not known initially. At the same time, the fluid viscosity and compressibility are known from laboratory studies of reservoir fluids; the working thickness of the reservoir is estimated on the basis of a well production log; and the porosity, permeability, and compressibility of the rock can be evaluated from results of the core examination. Using these known parameters, evaluation (5) allows an analysis of the time variation of the external radius for a certain permeability. An approximate evaluation (Fig. 2) showed that, for testing the well *a*, the external radius can be taken to be r_e = 250 m, since at a flow rate of 42 m^3/day (a maximum one in the test) and constant permeabilities of 0.127 and 0.106 μm^2 the depression funnel does not propagate beyond this radius for 1.2 day.* On the other hand, an error in the external radius, for example, as large as 50 m (20%), leads to a 3% error in the permeability evaluation (Fig. 2, permeability curve 3). It should be noted that the external radius adopted in interpreting the pressure-rate curve determines the value of the permeability alone, without influencing the pressure dependence of permeability (the permeability modulus is determined solely by the curve convexity).

Calculations indicated that the analytical solutions and the results of numerical simulation are in qualitative and quantitative agreement (Figs. 5 and 8). Calculations based on the dependence (Fig. 6, the well *a*, curve 1′), which was obtained using the method of [9] and the first point on the pressure-rate curve taken as the reference one (Fig. 5. the well *a*, i = 1), are in accord with measurements only in the first regime (Fig. 8, curve 1′), which is also observed in the analytical solution (Fig. 5, the well *a*, curve 1′). The dependence obtained using the proposed method (Fig. 6, the well *a*, curve 2′) provides an agreement between the pressure calculation and measurements in both regimes in analytical and numerical simulation (Fig. 5, the well *a*, curve 2′; Fig. 8, curve 2′). Thus, the proposed analytical procedure of solving the inverse and direct problems gives results that are in agreement with those of numeral simulation.

Conclusions. An integrated methodology of evaluating the variation in the reservoir permeability based on analytical and numerical simulation of an inflow to a well has been developed and approved. It includes the interpretation of well tests by analyzing both a quasi-steady-state inflow to a well (pressure-rate curves) and a transient response of pressure (pressure drawdown and buildup curves). The three components of the methodology are: 1) the detection of the permeability variation by comparing the pressure-drawdown and buildup curves in semi- and bilogarithmic coordinates, 2) the evaluation of the pressure dependence of permeability based on analytical solutions for a steady and unsteady-state inflow to a well, and 3) the approval and matching of the obtained dependence using numerical simulation of flows in a reservoir with varying permeability allow one to obtain the pressure dependence of permeability suitable for subsequent use in simulating the reservoir hydrodynamics without its additional matching.

This work was carried out with support from the Russian Foundation for Basic Research, grant OFI 07-01-97618.

NOTATION

B, volume coefficient of fluid; *c*, compressibility, MPa^{-1}; Ei, integral exponential [3, 13]; *f*, permeability modulus function; *h*, pay thickness of a reservoir, m; *I*, well productivity, $m^3 \cdot day^{-1} \cdot MPa^{-1}$; *k*, permeability, μm^2; *p*, pressure, MPa; $\bar{p}$, average pressure of a grid block, MPa; *Q*, well flow rate, $m^3 \cdot day^{-1}$; *q*, source flow density (well model), sec^{-1} (day^{-1}); *r*, radial coordinate, m; *S*, area under the flow curve; $\bar{S}$, area of the rectangle for the pressure-rate curve; *t*, time, sec (day); *x* and *y*, Cartesian coordinates, m; α, permeability modulus, MPa^{-1}; δ, accuracy of calculating the external radius; Δ, difference (drop), grid block size; ε, fluid conductivity, $m^3 \cdot mPa^{-1} \cdot sec^{-1}$; ϕ, porosity, fraction; κ, piezoconductivity, $m^2 \cdot sec^{-1}$; μ, dynamic viscosity, mPa·sec. Subscripts and superscripts: e, external radius of a well; *i*, point on the pressure-rate curve; f, fluid; p, productivity; r, rock; w, wall (bottom hole) of a well; Ω, boundary of the simulation area; 0, reference conditions.

REFERENCES

1. K. Aziz and A. Settari, *Mathematical Simulation of Reservoir Systems* [Russian translation], Nedra, Moscow (1982).
2. P. Quandalle and J. C. Sabathier, Typical features of a multipurpose reservoir simulator, *SPE Reservoir Engineering*, November, 1989, pp. 475–480.

*The values of constant permeability allow one to obtain flow rates at a corresponding depression on the considered pressure-rate curve (Fig. 5, the well *a*); only the largest value can be used in the analysis, since it will determine the maximum external radius.

3. V. N. Shchelkachev, *Principles and Applications of the Theory of Unsteady Filtration* [in Russian], Pts. 1, 2, Neft' i Gaz, Moscow (1995).
4. A. V. Raspopov and A. A. Shchipanov, Influence of dynamic deformation of a fractured-porous reservoir on oil production, *Neft. Khoz.*, No. 6, 97–98 (2002).
5. A. A. Shchipanov, Influence of dynamic deformation of a fractured-porous reservoir on the effect produced by an increase in depression, *Geol., Geofiz. Razrab. Neft. Gaz. Mestorozh.*, No. 2, 33–37 (2003).
6. R. G. Shagiev, *Well Tests Using Pressure Buildup Curves* [in Russian], Nauka, Moscow (1998).
7. Yu. M. Molokovich, A. I. Markov, A. A. Davletshin, and G. G. Kushtanova, *Piezometry of the Vicinity of Wells. Theoretical Principles* [in Russian], Izd. "DAS," Kazan' (2000).
8. D. Bourdet, *Well Test Analysis: the Use of Advanced Interpretation Models*, Elsevier (2002).
9. A. T. Gorbunov, *Development of Anomalous Oil Fields* [in Russian], Nedra, Moscow (1981).
10. V. D. Viktorin, A. F. Katoshin, and A. Yu. Nazarov, A geological field model of a bulk network of cracks (MBNC) of carbonate and sandstone collectors of fractured-porous type, in: *Problems of Geology and Development of Intricately Built Reservoirs of Fractured-Porous Type*, PermNIPIneft', Perm' (2003), pp. 60–117.
11. A. A. Shchipanov and A. Iu. Nazarov, Simulation of fluid flow in fractured reservoirs subject to deformation, *AAPG International Conference & Exhibition. Proceedings*, Paris, France (2005), pp. 1–6.
12. R. N. Diyashev, A. V. Kosterin, and É. V. Skvortsov, *Fluid Flow in Deformable Oil Reservoirs* [in Russian], Izd. Kazansk. Mat. Ob., Kazan' (1999).
13. K. S. Basniev, N. M. Dmitriev, and G. D. Rozenberg, *Oil-Gas Hydromechanics* [in Russian], Inst. of Computer Studies, Moscow–Izhevsk (2005).
14. D. K. Ponting, Corner point geometry in reservoir simulation, in: *Proc. Joint IMA/SPE European Conf. on the Mathematics of Oil Recovery*, July 1989, Cambridge (1989), pp. 1–8.
15. A. A. Shchipanov, Program package of modeling multiphase fluid filtration and the possibilities of its application, *Bulletin of Perm University. Information Systems and Technologies*, Issue 5, 108–114 (2001).
16. A. N. Tikhonov and V. Ya. Arsenin, *Methods of Solving Ill-Posed Problems* [in Russian], Nauka, Moscow (1979).
17. K. S. Basniev, I. N. Kochina, and V. M. Maksimov, *Underground Hydromechanics* [in Russian], Nedra, Moscow (1993).
18. N. P. Lebedinets, *Study and Development of Oil Fields with Fractured Reservoir* [in Russian], Nauka, Moscow (1997).
19. G. I. Marchuk, *Methods of Computational Mathematics* [in Russian], Nauka, Moscow (1980).